

# The Endogenous Price under Perfect Liquidity

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# Outline

- Introduction.
- The model.
- The endogenous price under perfect liquidity.
- The implications on liquidity measuring.
  - Revisited Liquidity-adjusted CAPM.
  - New bid-ask spread measure.
- Empirical investigation.
  - Liquidity-adjusted CAPM.
  - Bid-ask spread estimation.
- Conclusion.

# Conceptual Framework

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- Order-driven market  $\implies$  the placement of limit orders generates a winner's curse phenomenon.
- Biais, Glosten, and Spatt (2005) argue that liquidity suppliers placing limit orders behave as bidders in an auction.

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  - 1 Winner's curse risk
  - 2 Adverse selection risk ("picking-off" risk)
  - 3 Non-execution risk
- The trader placing a limit order is constrained to pay an implicit trading cost.

# Contributions to the Literature

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## Contributions to the Literature

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- 2 The model of the implicit trading cost curve  $\implies$  a valuation of the transaction cost caused by winner's curse under information asymmetry (winner's curse pricing model).
- The midquote conjecture  $\implies M = \frac{A+B}{2}$  is the approximation of the perfect liquidity price/fundamental value (Roll - 1984, JoF).

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- Thaler (1988, JEP) - the winner's curse phenomenon would not occur, if the bidders were perfectly rational.
- Garbarino and Slonim (2007, JRU) - the winner's curse is caused by information asymmetry.

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- Foucault, Kadan, and Kandel (2005) identify two types of traders, patient traders and impatient traders, and show that, at equilibrium, patient traders become liquidity suppliers for impatient traders.

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- Consider a continuous limit order market.
- There are two types of orders on this market: market orders and limit orders.
- There are two main categories of traders on the market: liquidity traders and liquidity suppliers. These traders are either informed or uninformed traders.
- The informed traders hold private information. The informed traders submit market orders. They have a cash-in-the-market strategy.
- There are also noise traders on the market, among both liquidity traders and liquidity suppliers.

# The Cash-in-the-Market Strategy

- The limit prices are

$$\begin{cases} K_b = E \left[ V | \Omega_t^a; \Theta^{lot}, \Gamma_b \right] \\ K_a = E \left[ V | \Omega_t^a; \Theta^{lot}, \Gamma_a \right] \end{cases}$$

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- The profit opportunity could occur in one of the following situations

$$\begin{cases} X \stackrel{not}{=} X_a > K_a \\ X \stackrel{not}{=} X_b < K_b \end{cases}$$

# The Replicating Option

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- **Sell limit order.** Example: A trader submits a sell limit order with the limit price of \$10. If the private information justifies a future price of \$11, the limit order offers to the informed traders a payoff of \$1.

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$$\text{Payoff} = \begin{cases} X_a - K_a, & \text{for sell limit order} \\ K_b - X_b, & \text{for buy limit order} \end{cases}$$

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- 4 If the replicating option is never exercised,

$$Payoff = 0$$

## Model Setup

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- *Assumption 1.* There is at least one informed trader on the market, holding the best possible estimate of the security's true value.
- *Assumption 2.* There is at least one uninformed liquidity supplier who will not cancel or modify the limit order, regardless of any new information arriving on the market.
- *Assumption 3.* In a risk-neutral world, the trading price follows a geometric Brownian motion

$$dP(t) = rP(t) dt + \sigma P(t) d\widetilde{W}(t)$$

## Model Setup (con't)

- *Assumption 4.* Let  $\tau_a$  be a hitting time which is defined as the first time when the process  $P(t)$  hits the value  $\bar{X}_a$ , where  $\bar{X}_a$  is a real positive number,  $\bar{X}_a > K_a$ , and  $P(t)$  is an adapted process with continuous paths

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- *Assumption 5.* Let  $\tau_b$  be a hitting time which is defined as the first time when the process  $P(t)$  hits the value  $\bar{X}_b$ , where  $\bar{X}_b$  is a real positive number,  $\bar{X}_b < K_b$ , and  $P(t)$  is an adapted process with continuous paths

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# Trading Cost Valuation

- Let  $K_b$  be the bid limit price of a buy limit order. The trading cost of the buy limit order caused by the winner's curse and the adverse selection is given by

$$\Pi_b = E_Q \left[ e^{-r\tau_b} (K_b - \bar{X}_b) \right] .$$

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- Let  $K_a$  be the ask limit price of a sell limit order. The trading cost of the sell limit order caused by the winner's curse and the adverse selection is given by

$$\Pi_a = E_Q \left[ e^{-r\tau_a} (\bar{X}_a - K_a) \right] .$$

# Winner's Curse Probability

## Proposition 1a (Winner's curse probability for a buy limit order).

When a buy limit order with the bid limit price  $K_b$  is executed, the probability that the winner's curse effect under information asymmetry does not occur is given by

$$Q(\tau_b = \infty) = \begin{cases} 1 - \left(\frac{\bar{X}_b}{K_b}\right)^{\frac{2r}{\sigma^2} - 1}, & \text{if } r > \frac{1}{2}\sigma^2 \\ 0, & \text{if } r \leq \frac{1}{2}\sigma^2 \end{cases}.$$

## Winner's Curse Probability (con't)

### Proposition 1b (Winner's curse probability for a sell limit order).

When a sell limit order with the ask limit price  $K_a$  is executed, the probability that the winner's curse effect under information asymmetry does not occur is given by

$$Q(\tau_a = \infty) = \begin{cases} 1 - \left(\frac{\bar{X}_a}{K_a}\right)^{\frac{2r}{\sigma^2} - 1}, & \text{if } r < \frac{1}{2}\sigma^2 \\ 0, & \text{if } r \geq \frac{1}{2}\sigma^2 \end{cases}.$$

# Trading Cost of a Buy Limit Order

## Proposition 2 (Trading cost of a buy limit order).

Let  $K_b$  be the bid limit price of a buy limit order. Let  $B$  be the quoted best bid at the order submission time. At the order submission time, the trading cost of the buy limit order is defined by

$$\Pi_b = \begin{cases} (K_b - \bar{X}_b) \frac{B}{\bar{X}_b}, & \text{if } \bar{X}_b \geq B \text{ and } K_b > B \\ (K_b - \bar{X}_b) \left( \frac{B}{\bar{X}_b} \right)^{-\frac{2r}{\sigma^2}}, & \text{if } \bar{X}_b < B \text{ and } K_b > B \text{ or } K_b \leq B \end{cases}.$$

# Trading Cost of a Buy Limit Order (con't)

## Corollary 1 (Maximum trading cost of a buy limit order).

Let  $K_b$  be the bid limit price of a submitted buy limit order. When  $\bar{X}_b < B$  the maximum trading cost of the buy limit order is defined by

$$\Pi_b^* = \frac{\sigma^2}{2r + \sigma^2} \left( \frac{2r}{2r + \sigma^2} \right)^{\frac{2r}{\sigma^2}} K_b^{\frac{2r + \sigma^2}{\sigma^2}} B^{-\frac{2r}{\sigma^2}}.$$

The optimum value of the bid price  $\bar{X}_b$  that maximizes the trading cost is given by

$$\bar{X}_b^* = \frac{2r}{2r + \sigma^2} K_b.$$

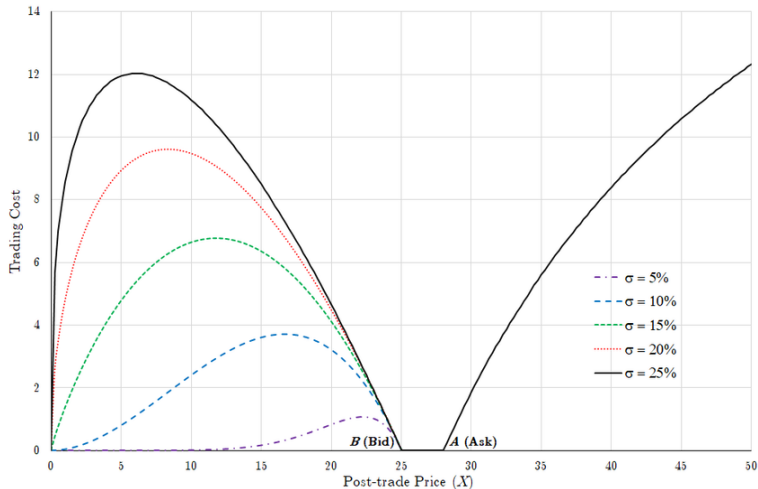
# Trading Cost of a Sell Limit Order

## Proposition 3 (Trading cost of a sell limit order).

Let  $K_a$  be the ask limit price of a sell limit order. Let  $A$  be the quoted best ask at the order submission time. At the order submission time, the trading cost of the sell limit order is defined by

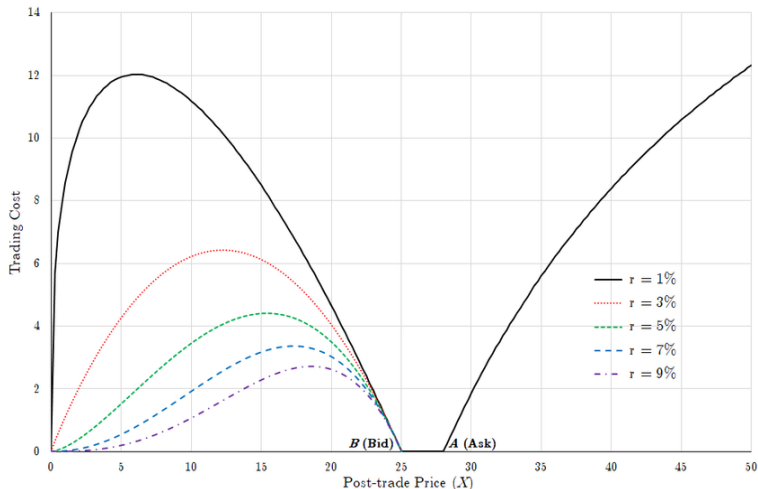
$$\Pi_a = \begin{cases} (\bar{X}_a - K_a) \frac{A}{\bar{X}_a}, & \text{if } \bar{X}_a \geq A \text{ and } A > K_a \\ (\bar{X}_a - K_a) \left( \frac{A}{\bar{X}_a} \right)^{-\frac{2r}{\sigma^2}}, & \text{if } \bar{X}_a < A \text{ and } A > K_a \text{ or } K_a \geq A \end{cases}.$$

# Trading Cost Curve Outside the Bid-Ask Spread

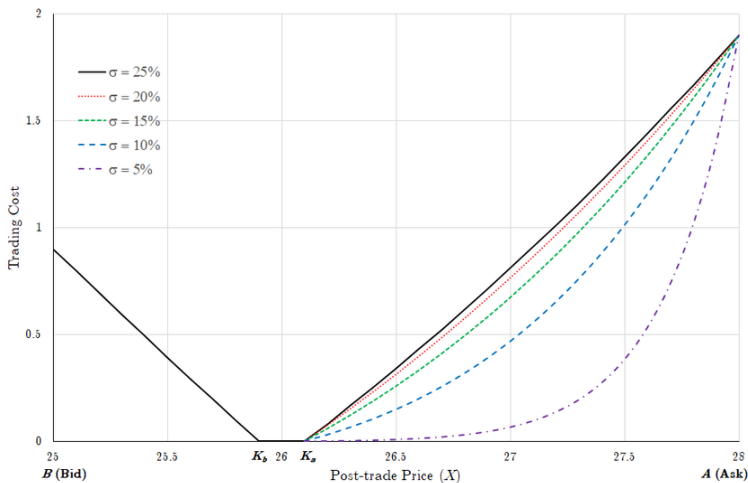




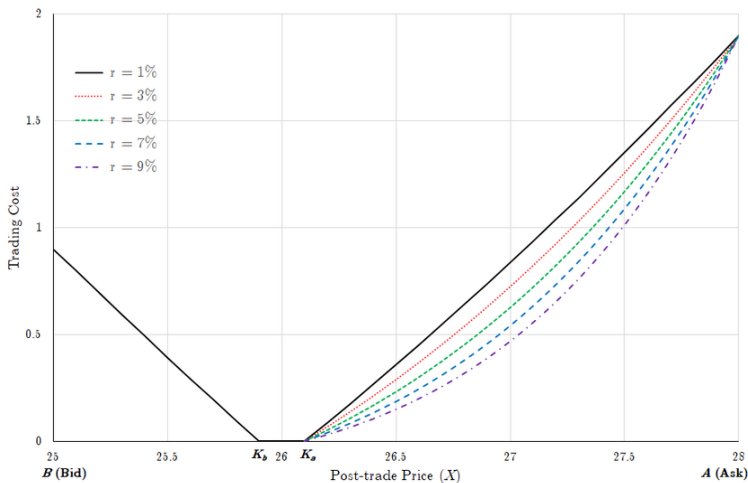
# Trading Cost Curve Outside the Bid-Ask Spread (con't)



# Trading Cost Curve Inside the Bid-Ask Spread



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- In a perfectly liquid market, the bid-ask spread is zero and the trading price is unique.  $L$  defines the perfect liquidity price. The price  $X_L$  is defined as the estimate of true value of the security under perfect liquidity condition:  $\bar{X}_a, \bar{X}_b \rightarrow X_L$ . The zero transaction cost condition is given by

$$(L - X_L) \frac{B}{X_L} + (X_L - L) \left( \frac{A}{X_L} \right)^{-\frac{2r}{\sigma^2}} = 0$$

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- When the value  $X_L$  approaches the perfect liquidity price ( $X_L \rightarrow L$ ), the illiquidity cost will tend to zero.

# The Perfect Liquidity Price (con't)

## Proposition 4 (Endogenous price under perfect liquidity).

Let  $A$  and  $B$  represent the best ask and the best bid prices in the order book. Let  $r$  be the risk-free interest rate and let  $\sigma$  be the volatility of the trading price. Then, the endogenous price under perfect liquidity is defined by

$$L = A^\gamma B^{1-\gamma},$$

where  $\gamma = r / (r + \frac{1}{2}\sigma^2)$  and  $0 < \gamma < 1$ .

# Endogenous Measures of Liquidity

- Revisited Quoted Spread (for small trades)

$$S_q = \frac{A - B}{L} = \left(\frac{A}{B}\right)^{1-\gamma} - \left(\frac{B}{A}\right)^{\gamma}$$



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- Revisited Weighted Quoted Spread (for large trades)

$$\bar{S}_q = \frac{\bar{A}(q) - \bar{B}(q)}{L}$$

## Endogenous Measures of Liquidity (con't)

- Revisited Effective Spread

$$S_e = \frac{|P - L|}{L}$$

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- Revisited Effective Spread at Bid Price

$$S_e^B = \frac{L - B}{L} = 1 - \left(\frac{B}{A}\right)^{\gamma}$$

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where  $R_{st} = \ln \frac{A_t}{B_t}$  is the return on spread at time  $t$ .

## Illiquidity Cost

- The net return is defined by

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where  $R_{st} = \ln \frac{A_t}{B_t}$  is the return on spread at time  $t$ .

- The relative illiquidity cost ( $C_{lt}$ ) is the difference between the gross return and the net return

$$C_{lt} = R_{lt} - R_t = \gamma_t R_{st} + (1 - \gamma_{t-1}) R_{st-1} > 0$$

## Revisited Liquidity-adjusted CAPM (LCAPM)

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- Using the endogenous relation between gross and net returns and CAPM relation,

$$E[R_{li} - C_{li}] = r + \beta_i (E[R_M] - r)$$

where

$$\begin{aligned}\beta_i &= \frac{\text{COV}(R_i, R_M)}{\text{VAR}(R_M)} = \frac{\text{COV}(R_{li} - C_{li}, R_M)}{\text{VAR}(R_M)} \\ &= \frac{\text{COV}(R_{li}, R_M)}{\text{VAR}(R_M)} - \frac{\text{COV}(C_{li}, R_M)}{\text{VAR}(R_M)} = \beta_{li} - \beta_{ci}\end{aligned}$$

## Revisited Liquidity-adjusted CAPM (LCAPM)

- Taking into consideration  $\beta_i$  coefficient decomposition and CAPM translation based on endogenous relation between gross and net returns, the expected gross return given by LCAPM is defined by

$$E[R_{li}] = r + \beta_{li} (E[R_M] - r) + E[C_{li}] - \beta_{ci} (E[R_M] - r)$$

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- The risk premium without liquidity risk is defined by

$$RP_i = \beta_{li} (E[R_M] - r)$$

## Bid-Ask Spread Measure

- Roll (1984, JoF), Stoll (1989, JoF), George, Kaul and Nimalendran (1991, RFS), Huang and Stoll (1997, RFS), Stoll (2000, JoF), Hasbrouck (2009, JoF), Corwin and Schultz (2012, JoF).

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- *Assumption 1.* The spread is constant over two consecutive trading periods (Corwin and Schultz),  $S = S_t = S_{t-1}$ .

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- *Assumption 1.* The spread is constant over two consecutive trading periods (Corwin and Schultz),  $S = S_t = S_{t-1}$ .
- *Assumption 2.* The net return is approximated by the effective return,  $R_t \simeq R_{et} = \ln \frac{P_t}{P_{t-1}}$ .

## Bid-Ask Spread Measure (con't)

### Proposition 5 (Bid-ask spread measure).

Let  $R_{et}$  be the effective return and let  $P_t$  be the trading price of the security. Let  $r_t$  be the risk-free interest rate and let  $\sigma_t$  be the volatility of the trading price. Then, the bid-ask spread of the security can be measured by

$$S = \left| \frac{E[R_{et}\Phi_t]}{E[\Phi_t^2]} \right|,$$

where  $R_{et} = \ln(P_t/P_{t-1})$  and  $\Phi_t = \gamma_t/P_t + (1 - \gamma_{t-1})/P_{t-1}$  with  $\gamma_t = r_t / (r_t + \frac{1}{2}\sigma_t^2)$ .



# Database

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- Risk-free interest rate: US 10-Year Treasury yield and Germany 10-year Treasury yield

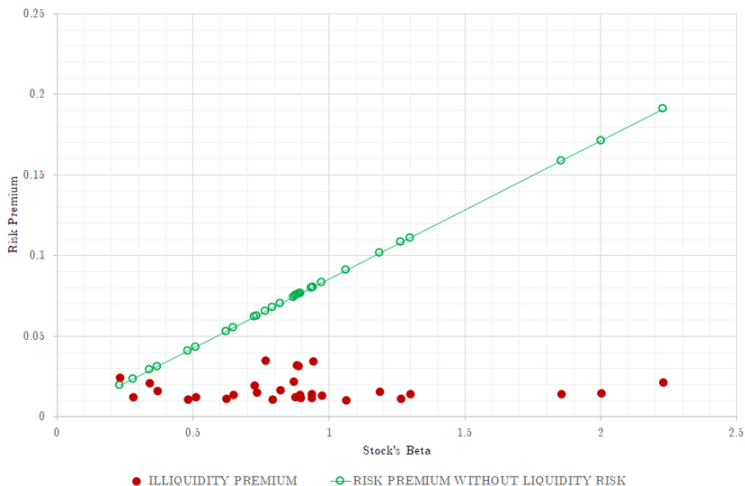
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- Stock Exchange: Euronext Paris and NYSE
- Period: May 2, 2016 - June 1, 2017
- Stock: 30 component stocks of DJIA and 30 component stocks of CAC40
- Risk-free interest rate: US 10-Year Treasury yield and Germany 10-year Treasury yield
- Variance is computed using the method of Parkinson (1980, JoB) and Garman and Klass (1980, JoB)

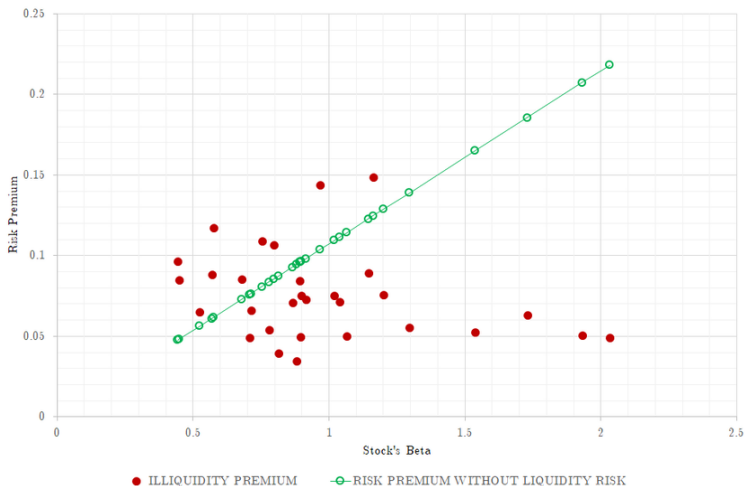
$$\sigma_t^2 = \frac{1}{\theta} \left[ \frac{1}{N} \sum_{i=t-N+1}^t \left( \ln \frac{H_i}{L_i} \right)^2 \right]$$

where  $\theta = 4 \ln 2$ .

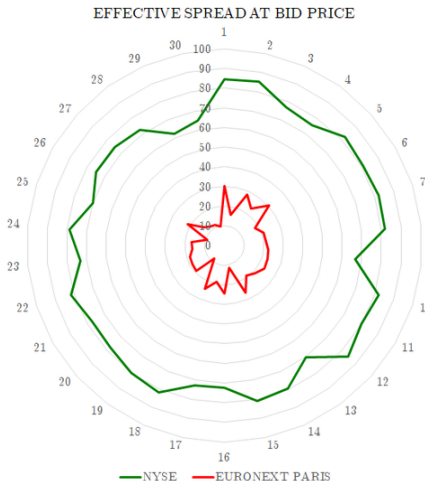
## Risk Premiums on NYSE



## Risk Premiums on Euronext Paris

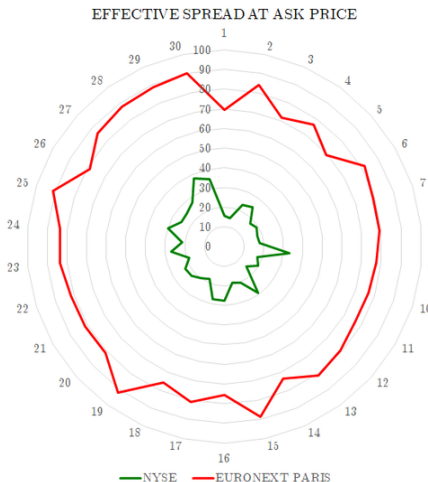


# Effective Spread Decomposition

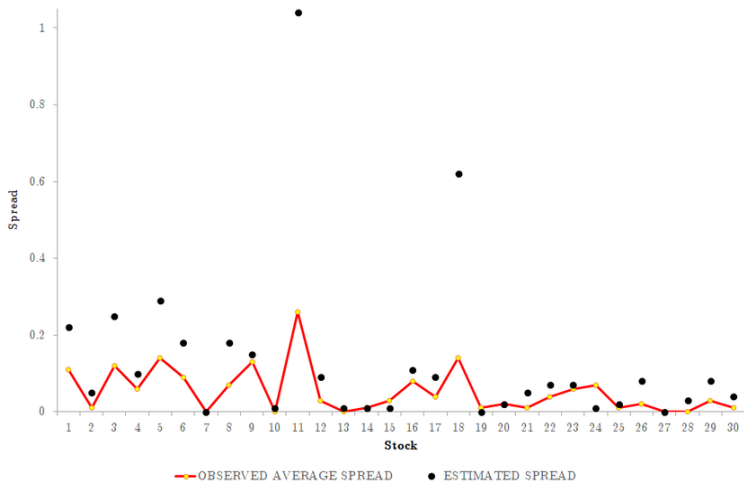




# Effective Spread Decomposition (con't)



## Estimated Spread vs. Observed Average Spread



## Conclusion

- I derive a model for evaluating the trading cost caused by the winner's curse phenomenon and the adverse selection effect.
- The main contribution of the article is the derivation of the endogenous price under perfect liquidity.
- The endogenous security price under perfect liquidity is a weighted average of the best bid and ask prices. The weights depend on the volatility of the security and on the risk-free interest rate.
- The perfect liquidity price is an alternative to the use of the mid-quote price in constructing various illiquidity measures.
- I redefine the quoted spread and the effective spread based on the price under perfect liquidity instead of the mid-quote price.

## Conclusion (con't)

- Through the liquidity-adjusted CAPM, I deduce the illiquidity premium by using the gross yield based on perfect liquidity prices and the net yield based on ask and bid prices.
- Based on the endogenous relationship between net and gross yields, I propose a new measure of the bid-ask spread.
- The empirical results indicate an impatient market enter behavior on the American market, as opposed to an impatient market exit behavior on the French market.